

# KATIE: for parton-level event generation with $k_T$ -dependent initial states

A. van Hameren

*Institute of Nuclear Physics Polish Academy of Sciences  
PL-31-342 Kraków, Poland*

## Abstract

KATIE is a parton-level event generator for hadron scattering processes that can deal with partonic initial-state momenta with an explicit transverse momentum dependence causing them to be space-like. Provided with the necessary transverse momentum dependent parton density functions, it calculates the off-shell matrix elements and performs the phase space importance sampling to produce weighted events, for example in the Les Houches Event File format. It can deal with arbitrary processes within the Standard Model, for up to four final-state particles and beyond. Furthermore, it can produce events for single-parton scattering as well as for multi-parton scattering.

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Formalism</b>	<b>4</b>
2.1	Event generation . . . . .	5
2.1.1	Importance sampling . . . . .	5
2.1.2	Un-weighting . . . . .	6
2.2	MPI . . . . .	6
<b>3</b>	<b>Usage</b>	<b>7</b>
3.1	Input file . . . . .	8
3.1.1	Processes . . . . .	8
3.1.2	PDFs . . . . .	9
3.1.3	Kinematics and cuts . . . . .	10
3.1.4	Model parameters . . . . .	11
3.1.5	Optimization . . . . .	12
3.2	Optimization stage . . . . .	12
3.3	Event generation . . . . .	13
<b>4</b>	<b>Summary</b>	<b>15</b>
<b>5</b>	<b>References</b>	<b>15</b>
<b>A</b>	<b>PDF sums</b>	<b>18</b>
<b>B</b>	<b>The meaning of <code>Esoft</code></b>	<b>19</b>

## 1 Introduction

Event generators are indispensable for the scientific activity in high energy physics. Scattering experiments performed at for example the Large Hadron Collider involve physical processes evolving at a large range of energy scales. Typical energy scales reach several tera electron volt in the so-called hard scattering process, in which heavy elementary particles like the Higgs boson are created, and go down to below the level of a giga electron volt when the eventual scattering remnants reach the detector. Multi purpose event generators [1–3] are designed to simulate scattering process at all those scales. The hard scattering process, which is computationally accessible with perturbative quantum chromodynamics (QCD), is object to dedicated studies, and programs designed for this purpose are called *parton-level event generators*. The descriptive/predictive power of either type of program is greatly enhanced by combining their efforts using so called merging and matching procedures [4].

The challenge for parton-level event generators is twofold: phase space points must be generated efficiently, and matrix elements must be evaluated quickly. Several incarnations of such programs exist, reaching NLO precision in QCD [5–7]. Much effort has gone in the efficient evaluation of one-loop amplitudes, and several programs have been developed for this sole purpose, which have been used in combination with SHERPA [3] to perform studies at NLO QCD [8–11]. All these programs function within a factorization prescription that separates the low-scale physics of the colliding protons from the high-scale physics of the hard process. More specifically, the cross section is a convolution of the parton density functions (PDFs) describing the protons, and the matrix element, describing the hard process. The PDFs only depend on the factorization scale and the fraction of the light-like proton momenta that enters the hard process, as prescribed by *collinear factorization* [12–14].

The initial-state partons in the hard process being parallel to the incoming hadrons is an approximation. For example, it implies that the final-state momenta can be separated into two groups that are back-to-back, which in a realistic collision is never the case. This approximation is traditionally cured by including higher fixed-order corrections, and/or by augmentation with a parton shower. It was argued in [15] that it would be desirable to remove this approximation already at lowest order in perturbation theory, and allow for non-vanishing transverse momentum components of the initial-state momenta in the hard process. The higher-order corrections may become much smaller, and it would open the possibility to include resummation corrections via transverse momentum dependent (TMD), or *un-integrated*, PDFs. An example of such an approach is *high-energy factorization* [16, 17].

On the side of the TMDs necessary for such an approach, there has been substantial activity [18]. Several evolution equations for TMDs have been developed, and the CCFM evolution [19] in particular is employed in the event generator CASCADE [20]. Several TMDs are provided by the library TMDLIB [21]. Besides the TMDs, such calculation requires matrix elements with off-shell initial-state partons. These cannot be obtained from collinear matrix elements by just changing the kinematics, because that would in general break gauge invariance. The problem of defining and calculating them, at least at tree-level, has been solved [22–27], and an automated implementation for arbitrary processes can be found in AVHLIB [28]. Furthermore, the program OGIME [29] can generate amplitudes with off-shell gluons.

Besides implementations of functions to evaluate the TMDs and the matrix elements, the calculation of a cross section or other observables requires Monte Carlo tools to perform the necessary phase space integrals. The program LXJET [30] can be used for several processes. For arbitrary processes, they are publicly available in AVHLIB [31, 32], and some calculations have already been performed [33–36],

Whereas AVHLIB was designed with the emphasis on versatility rather than on accessibility, KATIE constitutes a practical environment to produce event files using its tools. They can be produced in the Les Houches Event File (LHEF) format [37], or a custom format for which KATIE provides the tools to produce distributions of arbitrary observables. The definition of the process one would like to study happens via a single input file, containing all the information about which subprocesses are included, which PDFs are used, which cuts are applied, which values

of model parameters are used, etc.. KATIE does not provide any PDFs, but uses LHAPDF [38] for collinear PDFs, and automatically interpolates any TMD provided in the form of a (hyper-)rectangular grid. Finally, KATIE offers a convenient environment to perform calculations for multi-parton-scattering (MPI), in which more than one hard process occurs in each event simultaneously.

## 2 Formalism

We consider collision processes of specific hadrons resulting in a specific final state and refer to those with the symbol  $Y$ . The process  $Y = p p \rightarrow \mu^+ \mu^- j j$  (proton-proton to  $\mu^+ \mu^-$  plus two jets) is a typical example. Several partonic processes contribute in this example to which we refer with the symbol  $y$ , for example  $y = \bar{u} g \rightarrow \mu^+ \mu^- \bar{u} g$ . The separate particles in the partonic process are referred to by  $y_i$ . In the example given before, we have  $y_2 = g$  and  $y_4 = \mu^-$ . The number of final-state particles in  $Y$  is  $n = 4$  in the given example. The mass of final-state particle  $i$  is the same for every  $y \in Y$ .

A generally factorized formula for the differential cross section of a hadron collision process with a single parton-level scattering is given by

$$d\sigma_Y(p_1, p_2; k_3, \dots, k_{2+n}) = \sum_{y \in Y} \int d^4 k_1 \mathcal{P}_{y_1}(k_1) \int d^4 k_2 \mathcal{P}_{y_2}(k_2) d\hat{\sigma}_y(k_1, k_2; k_3, \dots, k_{2+n}) . \quad (1)$$

It is differential in the final-state momenta  $k_3, \dots, k_{2+n}$ . The symbols,  $p_1, p_2$  refer to the momenta of the hadrons, while  $k_1, k_2$  are the momenta of the initial-state partons. In collinear factorization, the distributions denoted with  $\mathcal{P}$  are given by

$$\mathcal{P}_{y_i}(k_i) = \int \frac{dx_i}{x_i} f_{y_i}(x_i, \mu) \delta^4(k_i - x_i p_i) , \quad (2)$$

where  $f_{y_i}$  is the collinear PDF for a parton of type  $y_i$  coming from hadron  $i$ , and  $\mu$  is the, possibly dynamical, factorization scale. For  $k_T$ -factorization the distributions typically look like

$$\mathcal{P}_{y_i}(k_i) = \int \frac{d^2 \mathbf{k}_{iT}}{\pi} \int \frac{dx_i}{x_i} \mathcal{F}_{y_i}(x_i, |\mathbf{k}_{iT}|, \mu) \delta^4(k_i - x_i p_i - k_{iT}) , \quad (3)$$

where now  $\mathcal{F}_{y_i}$  is the TMD or un-integrated PDF, and  $k_{iT}$  is the embedding of  $\mathbf{k}_{iT}$  in four-dimensional Minkowski space. The partonic differential cross section can be dissected further as follows:

$$d\hat{\sigma}_y(k_1, k_2; k_3, \dots, k_{2+n}) = d\Phi_Y(k_1, k_2; k_3, \dots, k_{2+n}) \Theta_Y(k_3, \dots, k_{2+n}) \times \text{flux}(k_1, k_2) \times \mathcal{S}_y |\mathcal{M}_y(k_1, \dots, k_{2+n})|^2 , \quad (4)$$

where  $d\Phi_Y$  is the  $(3n - 4)$ -dimensional differential phase space element for a final-state with  $n$  particles and masses dictated by  $Y$ :

$$d\Phi_Y(k_1, k_2; k_3, \dots, k_{2+n}) = (2\pi)^{4-3n} \left[ \prod_{i=3}^{2+n} d^4 k_i \delta_+(k_i^2 - m_i^2) \right] \delta^4(k_1 + k_2 - k_3 - \dots - k_{2+n}) . \quad (5)$$

The (squared) matrix element  $|\mathcal{M}_y|^2$  includes the summation over spins and colors of all external particles. It is turned into the average for the initial-state particles through the factor  $\mathcal{S}_y$ , which also includes the symmetry factor for the final state. The matrix elements used in KATIE are tree-level, and contain singularities which need to be avoided. This is established by phase space cuts  $\Theta_Y$ , typically consisting of minimum transverse momenta, maximum absolute rapidities, minimum distance between momenta in the two-dimensional space of rapidity and azimuthal angle ( $\Delta R$ ), etc.. The flux factor, finally, includes the demand that the energy of the partonic process is positive:

$$\text{flux}(k_1, k_2) = \frac{\theta((k_1 + k_2)^2)}{4\sqrt{(k_1 \cdot k_2)^2 - k_1^2 k_2^2}}. \quad (6)$$

The particular denominator is not prescribed by a specific factorization theorem, and is just an analytic continuation of the text book formula for massive initial-state particles. It is discussed further at the end of Section 3.1.3.

## 2.1 Event generation

KATIE creates event files consisting of a list of weighted phase space points such that

$$\frac{1}{N} \sum_{I=1}^N W_I \mathcal{B}(k_3^{(I)}, \dots, k_{2+n}^{(I)}) \approx \int d\sigma_Y(p_1, p_2; k_3, \dots, k_{2+n}) \mathcal{B}(k_3, \dots, k_{2+n}). \quad (7)$$

The sum is over the events, and the approximation formally becomes an equality when the number of events  $N$  goes to infinity.  $W_I$  is the weight of phase space point  $I$ , and  $k_j^{(I)}$  is the  $j$ -th momentum in event  $I$ .  $\mathcal{B}$  is a test function, for example the bin  $[z, z + \delta z]$  for a distribution of an observable  $\mathcal{O}$ :

$$\mathcal{B}(k_3, \dots, k_{2+n}) = \theta(\mathcal{O}(k_3, \dots, k_{2+n}) - z) \theta(z + \delta z - \mathcal{O}(k_3, \dots, k_{2+n})). \quad (8)$$

Of course, KATIE also provides  $k_1, k_2$  in the event file, and “observables” depending on these momenta can be studied too.

### 2.1.1 Importance sampling

KATIE uses importance sampling to reduce the fluctuation of the weights. Because of the multitude of possible PDFs and matrix elements causing the fluctuations, the importance sampling is partly achieved in an adaptive manner, and KATIE employs an optimization phase before it starts generating the actual events. For each partonic subprocess, a probability density  $P_y$  is adaptively created with the aim that

$$\mathcal{W}_y(k_1, \dots, k_{2+n}) = \frac{\mathcal{P}_{y_1}(k_1) \mathcal{P}_{y_2}(k_2) \text{flux}(k_1, k_2) \mathcal{S}_y |\mathcal{M}_y(k_1, \dots, k_{2+n})|^2}{P_y(k_1, \dots, k_{2+n})} \quad (9)$$

is as constant as possible. The differential cross section can be written as

$$d\sigma_Y = \sum_{y \in Y} dF_y(k_1, \dots, k_{2+n}) \Theta_Y(k_3, \dots, k_{2+n}) \mathcal{W}_y(k_1, \dots, k_{2+n}), \quad (10)$$

where we say that phase space points are generated following the distributions  $F_y$  given by

$$dF_y(k_1, \dots, k_{2+n}) = d^4k_1 d^4k_2 d\Phi_Y(k_1, k_2; k_3, \dots, k_{2+n}) P_y(k_1, \dots, k_{2+n}) . \quad (11)$$

Integration over  $k_1, k_2$  is understood in Eq. (10). The creation of the distributions  $F_y$  leads to crude estimates of the partonic sub cross sections

$$\sigma_y = \int dF_y(k_1, \dots, k_{2+n}) \Theta_Y(k_3, \dots, k_{2+n}) \mathcal{W}_y(k_1, \dots, k_{2+n}) . \quad (12)$$

During the event generation, first a subprocess is chosen with relative probability  $\sigma_y$ , and then a phase space point is generated following distribution  $F_y$ . If the event does not satisfy the phase space cuts  $\Theta_Y$ , it is rejected, and else it is accepted with weight

$$W = \mathcal{W}_y(k_1, \dots, k_{2+n}) \frac{\sum_{y' \in Y} \sigma_{y'}}{\sigma_y} . \quad (13)$$

So each event automatically has a subprocess assigned to it.

### 2.1.2 Un-weighting

Despite the importance sampling, the event weights may still fluctuate wildly, and supplementary techniques need to be applied to reduce this behavior. In the crude un-weighting method, the maximum weight  $W_{\max}$  is determined, and event  $I$  is accepted with probability  $W_I/W_{\max}$ . The main advantage is that all events get the same weight  $W_{\max}$ . The disadvantage is that in practice  $W_{\max}$  can only be determined within the total sample of events, and generating extra events may lead to a new increased  $W_{\max}$ , effectively reducing the total number of events.

KATIE uses partial un-weighting to reduce the weight fluctuation while avoiding the problem mentioned above. Using the first  $N_0$  events, a weight  $W_{\text{thrs}}$  is determined, and from then on, events with weight  $W_I \geq W_{\text{thrs}}$  are accepted and keep their weight, while events with weight  $W_I < W_{\text{thrs}}$  are accepted with probability  $W/W_{\text{thrs}}$ . If accepted, the latter get weight  $W_{\text{thrs}}$ . The threshold  $W_{\text{thrs}}$  is estimated with the aim to reach (the order of) a desired number of accepted events after reaching a desired statistical precision for the estimate of the total cross section. This way, a compromise can be chosen between accepting all events with wildly fluctuating weights, and accepting (possibly only very) few events with constant weight. More details can be found in the next section. It is important to mention that event files created with different values of  $W_{\text{thrs}}$  can be safely mixed without creating a bias, as long as they were created with *exactly the same set of distributions*  $F_y$ .

## 2.2 MPI

In the case of MPI, we consider the process  $Y$  to be separated into  $N_{\text{MPI}}$  processes  $Y^{(h)}$ . For example  $Y = p p \rightarrow \mu^+ \mu^- j j$  could be separated into  $Y^{(1)} = p p \rightarrow \mu^+ \mu^-$  and  $Y^{(2)} = p p \rightarrow j j$ . The hard processes are imagined to originate from one and the same pair of hadrons of course, but for our notation the above separation is most convenient.

KATIE treats MPI by simply factorizing all distributions, the matrix element, and the phase space. The only function that is not necessarily factorized is  $\Theta_Y$  representing the phase space cuts. Furthermore, there are extra restrictions requiring the sum of hadron momentum fractions coming from the same hadron to be smaller than one. The differential cross section becomes

$$d\sigma_Y = \frac{\mathcal{S}_{\text{MPI}}}{\sigma_{\text{eff}}} \prod_{h=1}^{N_{\text{MPI}}} \left[ \sum_{y \in Y^{(h)}} dF_y(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) \mathcal{W}_y(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) \right] \quad (14)$$

$$\times \Theta_Y(k_3 \dots, k_{2+n}) \theta \left( 1 - \sum_{h=1}^{N_{\text{MPI}}} x_1^{(h)} \right) \theta \left( 1 - \sum_{h=1}^{N_{\text{MPI}}} x_2^{(h)} \right),$$

where  $\sigma_{\text{eff}}$  is some phenomenologically determined normalization with units of a surface to the power  $N_{\text{MPI}} - 1$ , and  $\mathcal{S}_{\text{MPI}}$  is the symmetry factor associated with the MPI. For each set of, say  $l$ , identical  $Y^{(h)}$ , it contains a factor  $1/l!$ .

In the event generation, a subprocess  $y(h)$  is chosen for each  $Y^{(h)}$  with relative probability

$$\sigma_{y(h)} = \int dF_{y(h)}(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) \Theta_{Y^{(h)}}(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) \mathcal{W}_{y(h)}(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) . \quad (15)$$

These are estimated during the optimization of the distributions  $F_{y(h)}$ , which happens as if they concerned single-parton scattering processes, with cuts  $\Theta_{Y^{(h)}}$  that are chosen such that they cover enough phase space. Each event in the event file gets a weight

$$\frac{\mathcal{S}_{\text{MPI}}}{\sigma_{\text{eff}}} \prod_{h=1}^{N_{\text{MPI}}} \left[ \mathcal{W}_{y(h)}(k_1^{(h)}, \dots, k_{2+n_h}^{(h)}) \frac{\sum_{y' \in Y^{(h)}} \sigma_{y'}}{\sigma_{y(h)}} \right] . \quad (16)$$

The PDFs used to create the event file are of the single-parton type. The event file however provides the value of  $\mathcal{P}_{y_1(h)} \mathcal{P}_{y_2(h)}$  for each  $Y^{(h)}$ , and parton correlations can be included by multiplying the event weight with

$$\frac{\mathcal{D}(k_1^{(1)}, k_2^{(1)}; \dots; k_1^{(N_{\text{MPI}})}, k_2^{(N_{\text{MPI}})})}{\prod_{h=1}^{N_{\text{MPI}}} \mathcal{P}_{y_1(h)}(k_1^{(h)}) \mathcal{P}_{y_2(h)}(k_2^{(h)})} , \quad (17)$$

where  $\mathcal{D}$  is the desired (very general) multi-parton correlation function.

### 3 Usage

KATIE requires a Fortran 2003 compiler (implemented at least as far as in gfortran-4.6), Python, and Bash. The program can be obtained from

<http://bitbucket.org/hameren/katie/downloads>

For the following, we refer to the directory where KATIE is installed, that is the directory that

contains the file `settings.py`, with `$KaTie`. Before KATIE can be used, the user must download AVHLIB, which can be obtained from

<http://bitbucket.org/hameren/avhlib/downloads>

We refer to the directory where AVHLIB is installed with `$AVHLIB`. Next, the user must edit the file `$KaTie/settings.py`, and set the path to the AVHLIB-directory (so the value of `$AVHLIB`), the path to the LHAPDF-directory (the directory where the library file is), and the Fortran compiler. Then, the user must execute

```
$ $KaTie/config.py lib
```

in order to create a library. The easiest way to proceed is to choose an example in the directory `$KaTie/examples` that is closest to the user's wishes, and use this as a starting point. Let us say it is `pp_to_4j`. The user must choose a non-existent directory, from now on referred to as `$project`, and execute

```
$ $KaTie/work.sh pp_to_4j $project
```

This will create `$project` with some files and a symbolic link to the run-script inside. In this example, there are two input files: one for single-parton scattering (SPS) and one for double-parton scattering (DPS).

### 3.1 Input file

The input file consist of lines with the structure `keyword = value`. It is essential to keep at least one space (like between `keyword`, `=`, and `value` above) whenever there is one in the example input file, or in the following. We will continue with the DPS, because it requires slightly more explanation.

#### 3.1.1 Processes

The first parameter in `$projects/input_dps`

```
Ngroups = 2
```

sets the number of scattering groups. This is 1 for SPS, 2 for DPS etc.. In this case the number of final-state particles must be set for each of these, thus the 2 numbers on the right in the line

```
Nfinst = 2 2
```

Next, the contributing processes are set, for example with the line

```
process = q q~ -> r r~ factor = Nf-1 groups = 1 2 pNonQCD = 0 0 0
```

It refers to the parton-level process  $q\bar{q} \rightarrow q'\bar{q}'$ . Using the symbols `q` and `r` to refer to quarks implies summation over initial-state combinations of quarks that have the same matrix element within pure QCD. This would be incorrect if electro-weak interactions are involved, and then the quarks should be denoted `u` for up-type quarks and `d` for down-type quarks. Then there is still a



summation over combinations that have equal matrix elements. All combinations are explicitly given in Appendix A. If no summation is desired, then the parameter

```
partlumi = individual
```

must be set. Summation over final-state quarks with equal matrix elements is achieved via the `factor` in the process-line. The next parameter, `groups`, indicates to which groups the process contributes. In the case of DPS for 4 jets, all  $2 \rightarrow 2$  processes contribute both groups, but for the production of  $\mu^+\mu^-jj$  for example this would not be the case. The last parameter in the process line, `pNonQCD`, indicates the power of the electro-weak coupling, the Higgs-gluon coupling, and the Higgs-photon coupling respectively. This is relevant again for example for  $pp \rightarrow \mu^+\mu^-jj$ , to which both  $\mathcal{O}(\alpha_s^2\alpha_{EW}^2)$  and  $\mathcal{O}(\alpha_{EW}^4)$  Feynman graphs contribute, while one would like to exclude the latter. The names of all possible particles are

```
ve  ve~  e-   e+   u  u~  d  d~
vmu vmu~ mu-  mu+  c  c~  s  s~
vtau vtau~ tau- tau+ t  t~  b  b~
g  H  A  Z  W+  W-
```

They should all be obvious, except maybe the neutrinos which start with `v`, and the photon `A`.

### 3.1.2 PDFs

The next parameter sets the pdf set from LHAPDF, for example

```
lhaSet = MSTW2008nlo68cl
```

It needs to be set also for off-shell initial-state partons, because KATIE takes  $\alpha_s$  from there. The line

```
offshell = 0 0
```

determines that no partons are off-shell. The other possibilities are `0 1`, `1 0`, and `1 1`. KATIE does not provide any  $k_T$ -dependent pdfs. They can, for example, be obtained from TMDLIB

<https://www.hepforge.org/downloads/tmdlib/>

The data files provided by TMDLIB need to be converted in order to be used in KATIE, which can be achieved with the script `$KaTie/tmdconvert.sh`. The command

```
$ $KaTie/tmdconvert.sh
```

prints further usage on the screen. The result of this conversion are files consisting of three or four columns, containing

$$\ln(x) \quad \ln(|\mathbf{k}_T|^2) \quad xf(x, |\mathbf{k}_T|)$$

or

$$\ln(x) \quad \ln(|\mathbf{k}_T|^2) \quad \ln(\mu^2) \quad xf(x, |\mathbf{k}_T|, \mu)$$

if the pdf also depends on the hard scale. The hard scale dependence does not have to be indicated anywhere, and is recognized automatically. KATIE interpolates the grids herself. The directory

where KATIE can find the grids is indicated in the input file by

```
tmdTableDir = /home/user/projects/tmdgrids/
```

where here of course an example path is given. The actual grid file must be indicated for each parton separately with

```
tmdpdf = g gluon.dat  
tmdpdf = u uQuark.dat  
tmdpdf = u~ uBar.dat
```

etc., where the file names are examples again. The path can be changed (or rather updated) between lines indicating the files, in case the files are distributed over several directories. In the case of  $k_T$ -factorization, a TMD for the gluon *must always* be provided, also if the user happens to only want to study quark-initiated processes. The number of active flavors is set with

```
Nflavors = 5
```

and in case of DPS, the value of  $\sigma_{\text{eff}}$  is set with

```
sigma_eff = 15d6
```

in units of nanobarn.

### 3.1.3 Kinematics and cuts

The center-of-mass energy of the scattering is set in GeV with

```
Ecm = 7000
```

The phase space pre-sampler needs information about the typical value of the softest scale, like for example a minimum  $p_T$ , which is set in GeV with

```
Esoft = 20
```

This number *is not a cut-off* and only influences the efficiency of the optimization, as explained in Appendix B. It must be larger than zero. The actual phase space cuts are set explicitly in the input file.

```
cut = {deltaR|2,4|} > 0.5
```

sets the minimum value of  $\Delta R$  between final-state particle 2 and 4. At tree-level, the level of KATIE, this corresponds the value of  $R$  in a jet algorithm. These numbers refer to the order as given in the process list.

```
cut = {pT|2|1,2,3,4} > 50
```

sets the minimum  $p_T$  in GeV for the second final-state in the  $p_T$ -ordered list of final state number 1, 2, 3, and 4. A minimum  $p_T$  for final state 2 directly can be set with  $\{p_T|2|\}$ , and a maximum  $p_T$  can be set with  $<$ .

```
cut = {mass|1+3+4|} > 100
```

sets a minimum for the invariant mass of the sum of final-state momenta 1, 3, and 4. Similarly, other variables also can take arguments that consist of sums, *e.g.* the  $p_T$  of the sum of final-state momenta 2 and 3 is  $\{p_T|2+3|\}$ . Other possible variables are `rapidity`, `pseudoRap`, `ET` (also called the transverse mass), `deltaPhi`. The meaning of the variables should be obvious. The value of `rapidity` and `pseudoRap` can be negative, so both the minimum and maximum must be set explicitly. On the other hand, `deltaPhi` is positive between 0 and  $\pi$ .

The hard scale that goes into the pdfs and  $\alpha_s$  is set with for example

```
scale = ( {pT|1| } + {pT|2| } + {pT|3| } + {pT|4| } ) / 2
```

No spaces are allowed in the expression on the right-hand-side. All functions mentioned before with the cuts are allowed. Numerical constants and the arithmetic operations  $+$ ,  $-$ ,  $*$ ,  $/$ ,  $**$ , as well as parentheses are allowed. In the case of DPS, one would like to have separate scales for the different groups. This can be achieved with

```
scale = entry 1 ( {pT|1| } + {pT|2| } ) / 2
scale = entry 2 ( {pT|3| } + {pT|4| } ) / 2
```

In the case of DPS, cuts and scale also have to be given for each group separately. To set these, the lines in the input file need, for example for group 1, to look like

```
cut = group 1 {pT|2|1,2,3,4} > 50
scale = group 1 ( {pT|1| } + {pT|2| } ) / 2
```

etc.. They are only necessary for optimization purposes.

As mentioned earlier, the denominator of the flux factor in Eq. (6) is not prescribed by a specific factorization theorem. It is argued in [39] that within the framework of Kimber, Martin, and Ryskin [40] the appropriate denominator is the generalization of the one in collinear factorization, so

$$\text{flux}(k_1, k_2) = \frac{\theta((k_1 + k_2)^2)}{8k_1^0 k_2^0} . \quad (18)$$

This is the default in KATIE. The one of Eq. (6) can be set with the option

```
flux factor = textbook
```

### 3.1.4 Model parameters

Particle masses and (fixed) widths are set with

```
mass = Z 91.1882 2.4952
mass = W 80.419 2.21
mass = H 125.0 0.00429
mass = t 173.5
```

Other particles are massless by default, but can be given a mass too. The user can indicate which interactions are active with the lines

```

switch = withQCD Yes
switch = withQED No
switch = withWeak No
switch = withHiggs No
switch = withHG No
switch = withHA No

```

where the last two refer to the effective Higgs-gluon and Higgs-photon interactions. These and withHiggs are switched off by default, while the others are switched on. The electro-weak coupling is fixed, and can be set with for example

```
coupling = alphaEW 0.00794
```

Alternatively, the value of Fermi's constant  $G_F$  can be set with for example

```
coupling = Gfermi 1.16639d-5
```

The electro-weak coupling is then set to  $\alpha_{EW} = G_F \frac{\sqrt{2}}{\pi} m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)$ .

### 3.1.5 Optimization

If there are many final-state particles, the sum over helicities becomes unnecessarily time consuming, and the user should choose to sample over helicities instead:

```
helicity = sampling
```

Other possible values are sum and polarized. The latter chooses the continuous sampling method of [41]. The number of events to be spent on the optimization of the pre-sampler also needs to be set in the input file.

```
Noptim = 100,000
```

sets the number of non-vanishing-weight events to a hundred thousand. This will be discussed in more detail in the following.

## 3.2 Optimization stage

Event generation happens in two stages. During the first stage, the pre-sampler is optimized for each process given in the input file separately. Executing

```
$ $project/run.sh prepare $project/input_sps $project/trial01
```

will create a directory `$project/trial01` and inside a directory will be created for each process given in the input file `input_sps`. The name `trial01` is just an example. Executing

```
$ $project/trial01/optimize.sh
```

will start the optimization of all processes. If there are very many processes, one might want the optimization to happen in batches, and by executing for example

```
$ $project/trial01/optimize.sh Ncpu=4
```

only 4 optimizations will run simultaneously until all have been performed. This number does not have to correspond to the actual number of CPUs. One can also select processes to be optimized, *e.g.*

```
$ $project/trial01/optimize.sh Ncpu=4 proc=1,7,13,24,25
```

The progress can be monitored by viewing the output files in each process directory, for example with

```
$ tail -n6 $project/trial01/proc*/output
```

The final precision should not be more than a few percent. For example

```
MESSAGE from Kaleu stats: Ntot = 38,887
```

```
MESSAGE from Kaleu stats: + 25,600 (.13072883+/- .00132728)E+06 1.015
```

means that 38887 events were generated, of which 25600 passed the cuts, leading to an estimated cross section of  $0.1307 \times 10^6 \text{nb}$  with an estimated statistical error of 1.015%. The + in front of 25,600 means that it concerns positive-weight events. In case the optimization of a process does not seem to converge, the user can try to increase the number of events for that process and/or change the random number seed. Say the user wants to re-run process number 3 with seed=273846 and with Noptim=200000. Then the user needs to execute

```
$ prefix=$project/trial01/proc03
```

```
$ $prefix/main.out seed=273846 Noptim=200000 > $prefix/output &
```

Alternatively, the user can edit the appropriate lines in `$project/trial/optimize.sh`. For any seed, the result of the optimization will be an unbiased, but not necessarily efficient, phase space pre-sampler. The seed can be different for every process. The only rule is that once you start to generate event files, the pre-sampler must be fixed, and you **MUST NOT** re-run the optimization.

### 3.3 Event generation

The second stage is the actual event generation, and happens after the optimization is finished. The user may just execute

```
$ prefix=$project/trial01
```

```
$ nohup $prefix/main.out seed=732415 > $prefix/output1 &
```

```
$ nohup $prefix/main.out seed=232984 > $prefix/output2 &
```

etc. for several different random number seeds. So-called “raw” event files will be produced in `$project/trial01` with the names `raw732415.dat` `raw232984.dat` etc.. They contain weighted events that form a partly un-weighted collection of events from all that are generated by the pre-sampler. The number of events and the rate of fluctuation of their weights can be steered to some degree, as explained in Section 2. By default, in the order of  $10^5$  events are accepted when the cross section is estimated to a statistical precision of  $0.001 = 0.1\%$ . These numbers

can be changed by providing optional arguments. For example

```
$ nohup $prefix/main.out Nevents=1e6 precision=0.01 seed=732415 \
> $prefix/output1 &
```

will try to accept in the order of  $10^6$  events until a statistical precision of  $0.01 = 1\%$  is reached. In this case, of course, the events will be of “lower quality” in the sense that their weights will fluctuate more.

In order to create a LHEF `$project/trial01/eventfile.dat`, the user must execute

```
$ $project/run.sh lhef $project/trial01/raw*
```

This will use all available raw files. The user may also select some by listing them separately in the above command. This command may be executed before the event generations have finished. Alternatively, in order to create an event file that can be processed with KATIE further, the user may execute

```
$ $project/run.sh merge $project/trial01/raw*
```

An ascii file `$project/trial01/eventfile.dat` will be created. After obvious information in the header, the events are listed. Each event block starts with, for example

```
EVENT WEIGHT: 0.5778970106138136E+09
```

where the floating point number is the weight value  $W$  assigned to the event. The weights are normalized such that

$$\frac{\sum_{\text{events}} W}{\sum_{\text{events}} 1} = \text{total cross-section}.$$

The next line contains a single integer, referring to the process corresponding to the event. The process numbering is given in the header of the event file. The next lines consist of 5 floating point numbers and 2 integers, containing the momenta and the color flow of the event:

```
E      px      py      pz      E2 - px2 - py2 - pz2      color      anti-color
```

Initial-state momenta have a negative value of the energy  $E$ . Then follows a line consisting of 4 floating point numbers, containing the value of

```
matrix element      parton luminosity      αs      scale
```

The matrix element is not averaged regarding the initial-state partons. The parton luminosity is just the product of the pdfs. For multi-parton scattering, event blocks are repeated for a single event weight.

In order to make histograms, the user may use the Fortran program `read_event_file.f90` in the directory `$project` as a starting point. It can be compiled and executed with

```
$ $project/run.sh compile $project/read_event_file.f90
$ $project/read_event_file.out $project/trial01/eventfile.dat
```

The program uses the Fortran module `read_events_mod` which provides several variables,

functions and subroutines, as well as two types to create one-dimensional and two-dimensional histograms. They are listed at the beginning of `read_event_file.f90`.

## 4 Summary

KATIE, a program for parton-level generation of events with initial-states that can have non-vanishing transverse momentum components, was presented. It provides all necessary ingredients, including off-shell matrix elements and an efficient importance sampler, except the transverse momentum dependent parton density functions. The latter can be provided in the form of hyper-rectangular grids which KATIE will automatically interpolate. Events are produced in the Les Houches Event File format, or in a custom format with which distributions of arbitrary observables can be produced with tools also provided by KATIE. Finally, a convenient environment is available to perform calculations involving multi-parton interactions.

## Acknowledgments

The author would like to thank Marcin Bury, Hannes Jung, Mirko Serino, and Krzysztof Bożek for their help in beta testing the program. Also, the author would like to thank Maxim Nefedov and Vladimir Saleev for valuable feedback. This work was supported by grant of National Science Center, Poland, No. 2015/17/B/ST2/01838.

## 5 References

- [1] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, *An Introduction to PYTHIA 8.2*, *Comput. Phys. Commun.* **191** (2015) 159–177, [1410.3012].
- [2] J. Bellm *et al.*, *Herwig++ 2.7 Release Note*, 1310.6877.
- [3] T. Gleisberg, S. Hoeche, F. Krauss, M. Schonherr, S. Schumann, F. Siegert, and J. Winter, *Event generation with SHERPA 1.1*, *JHEP* **02** (2009) 007, [0811.4622].
- [4] T. Sjöstrand, *Status and developments of event generators*, in *4th Large Hadron Collider Physics Conference (LHCP 2016) Lund, Sweden, June 13-18, 2016*, 2016. 1608.06425.
- [5] G. Bevilacqua, M. Czakon, M. V. Garzelli, A. van Hameren, A. Kardos, C. G. Papadopoulos, R. Pittau, and M. Worek, *HELAC-NLO*, *Comput. Phys. Commun.* **184** (2013) 986–997, [1110.1499].
- [6] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, *The automated computation of tree-level and*

- next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079, [1405.0301].
- [7] C. Weiss, B. Chokoufe Nejad, W. Kilian, and J. Reuter, *Automated NLO QCD Corrections with WHIZARD*, *PoS EPS-HEP2015* (2015) 466, [1510.02666].
  - [8] S. Badger, A. Guffanti, and V. Yundin, *Next-to-leading order QCD corrections to di-photon production in association with up to three jets at the Large Hadron Collider*, *JHEP* **03** (2014) 122, [1312.5927].
  - [9] H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, and T. Peraro, *Next-to-Leading-Order QCD Corrections to Higgs Boson Production in Association with a Top Quark Pair and a Jet*, *Phys. Rev. Lett.* **111** (2013), no. 17 171801, [1307.8437].
  - [10] Z. Bern, L. J. Dixon, F. Febres Cordero, S. Hoeche, H. Ita, D. A. Kosower, N. A. Lo Presti, and D. Maitre, *Next-to-leading order  $\gamma\gamma + 2$ -jet production at the LHC*, *Phys. Rev.* **D90** (2014), no. 5 054004, [1402.4127].
  - [11] S. Kallweit, J. M. Lindert, P. Maierhofer, S. Pozzorini, and M. Schönherr, *NLO QCD+EW predictions for  $V + \text{jets}$  including off-shell vector-boson decays and multijet merging*, *JHEP* **04** (2016) 021, [1511.08692].
  - [12] S. B. Libby and G. F. Sterman, *Jet and Lepton Pair Production in High-Energy Lepton-Hadron and Hadron-Hadron Scattering*, *Phys. Rev.* **D18** (1978) 3252.
  - [13] R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, *Perturbation Theory and the Parton Model in QCD*, *Nucl. Phys.* **B152** (1979) 285–329.
  - [14] J. C. Collins, D. E. Soper, and G. F. Sterman, *Factorization of Hard Processes in QCD*, *Adv. Ser. Direct. High Energy Phys.* **5** (1989) 1–91, [hep-ph/0409313].
  - [15] J. Collins and H. Jung, *Need for fully unintegrated parton densities*, in *HERA and the LHC: A Workshop on the implications of HERA for LHC physics. Proceedings, Part B*, 2005. hep-ph/0508280.
  - [16] S. Catani, M. Ciafaloni, and F. Hautmann, *High-energy factorization and small  $x$  heavy flavor production*, *Nucl. Phys.* **B366** (1991) 135–188.
  - [17] J. C. Collins and R. K. Ellis, *Heavy quark production in very high-energy hadron collisions*, *Nucl. Phys.* **B360** (1991) 3–30.
  - [18] R. Angeles-Martinez *et al.*, *Transverse Momentum Dependent (TMD) parton distribution functions: status and prospects*, *Acta Phys. Polon.* **B46** (2015), no. 12 2501–2534, [1507.05267].



- [19] S. Catani, F. Fiorani, and G. Marchesini, *Small  $x$  Behavior of Initial State Radiation in Perturbative QCD*, *Nucl. Phys.* **B336** (1990) 18–85.
- [20] H. Jung *et al.*, *The CCFM Monte Carlo generator CASCADE version 2.2.03*, *Eur. Phys. J.* **C70** (2010) 1237–1249, [1008.0152].
- [21] F. Hautmann, H. Jung, M. Krämer, P. J. Mulders, E. R. Nocera, T. C. Rogers, and A. Signori, *TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions*, *Eur. Phys. J.* **C74** (2014) 3220, [1408.3015].
- [22] L. N. Lipatov, *Gauge invariant effective action for high-energy processes in QCD*, *Nucl. Phys.* **B452** (1995) 369–400, [hep-ph/9502308].
- [23] L. N. Lipatov and M. I. Vyazovsky, *QuasimultiRegge processes with a quark exchange in the  $t$  channel*, *Nucl. Phys.* **B597** (2001) 399–409, [hep-ph/0009340].
- [24] E. N. Antonov, L. N. Lipatov, E. A. Kuraev, and I. O. Cherednikov, *Feynman rules for effective Regge action*, *Nucl. Phys.* **B721** (2005) 111–135, [hep-ph/0411185].
- [25] A. van Hameren, P. Kotko, and K. Kutak, *Helicity amplitudes for high-energy scattering*, *JHEP* **01** (2013) 078, [1211.0961].
- [26] A. van Hameren, K. Kutak, and T. Salwa, *Scattering amplitudes with off-shell quarks*, *Phys. Lett.* **B727** (2013) 226–233, [1308.2861].
- [27] P. Kotko, *Wilson lines and gauge invariant off-shell amplitudes*, *JHEP* **07** (2014) 128, [1403.4824].
- [28] M. Bury and A. van Hameren, *Numerical evaluation of multi-gluon amplitudes for High Energy Factorization*, *Comput. Phys. Commun.* **196** (2015) 592–598, [1503.08612].
- [29] P. Kotko, OGIME, <http://nz42.ifj.edu.pl/~pkotko/ogime.html>
- [30] P. Kotko, LxJet, <http://nz42.ifj.edu.pl/~pkotko/lxjet.html>
- [31] A. van Hameren, *PARNI for importance sampling and density estimation*, *Acta Phys. Polon.* **B40** (2009) 259–272, [0710.2448].
- [32] A. van Hameren, *Kaleu: A General-Purpose Parton-Level Phase Space Generator*, 1003.4953.
- [33] K. Kutak, R. Maciuła, M. Serino, A. Szczurek, and A. van Hameren, *Search for optimal conditions for exploring double-parton scattering in four-jet production:  $k_T$ -factorization approach*, *Phys. Rev.* **D94** (2016), no. 1 014019, [1605.08240].

- [34] K. Kutak, R. Maciuła, M. Serino, A. Szczurek, and A. van Hameren, *Four-jet production in single- and double-parton scattering within high-energy factorization*, *JHEP* **04** (2016) 175, [1602.06814].
- [35] A. van Hameren, P. Kotko, and K. Kutak, *Resummation effects in the forward production of  $Z_0$ +jet at the LHC*, *Phys. Rev.* **D92** (2015), no. 5 054007, [1505.02763].
- [36] A. van Hameren, R. Maciuła, and A. Szczurek, *Production of two charm quark-antiquark pairs in single-parton scattering within the  $k_t$ -factorization approach*, *Phys. Lett.* **B748** (2015) 167–172, [1504.06490].
- [37] J. Alwall *et al.*, *A Standard format for Les Houches event files*, *Comput. Phys. Commun.* **176** (2007) 300–304, [hep-ph/0609017].
- [38] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, *LHAPDF6: parton density access in the LHC precision era*, *Eur. Phys. J.* **C75** (2015) 132, [1412.7420].
- [39] M. Nefedov and V. Saleev, *Diphoton production at the Tevatron and the LHC in the NLO approximation of the parton Reggeization approach*, *Phys. Rev.* **D92** (2015), no. 9 094033, [1505.01718].
- [40] M. A. Kimber, A. D. Martin, and M. G. Ryskin, *Unintegrated parton distributions and prompt photon hadroproduction*, *Eur. Phys. J.* **C12** (2000) 655–661, [hep-ph/9911379].
- [41] P. Draggiotis, R. H. P. Kleiss, and C. G. Papadopoulos, *On the computation of multigluon amplitudes*, *Phys. Lett.* **B439** (1998) 157–164, [hep-ph/9807207].
- [42] F. James, *RANLUX: A FORTRAN implementation of the high quality pseudorandom number generator of Luscher*, *Comput. Phys. Commun.* **79** (1994) 111–114. [Erratum: *Comput. Phys. Commun.* 97,357(1996)].

## A PDF sums

In the following, the indices 1, 2 refer to initial-state 1 and 2. Each left-hand-side below implies the sum on the right-hand-side, because for each term on the right-hand-side the numerical value of the matrix element is identical. We consider the 5-flavor case. For the 4-flavor case  $b$  and  $\bar{b}$  quarks are removed, for the 3-flavor case also  $c$  and  $\bar{c}$  quarks are removed, and for the 2-flavor case also  $s$  and  $\bar{s}$  quarks are removed. In case of no electro-weak interactions, there are the following initial-state cases, which all need to be included for example for the processes

pp  $\rightarrow$  jets:

$$g \ g : g_1 g_2 \quad (19)$$

$$g \ q : g_1(u_2 + \bar{u}_2 + c_2 + \bar{c}_2 + d_2 + \bar{d}_2 + s_2 + \bar{s}_2 + b_2 + \bar{b}_2) \quad (20)$$

$$q \ g : (u_1 + \bar{u}_1 + c_1 + \bar{c}_1 + d_1 + \bar{d}_1 + s_1 + \bar{s}_1 + b_1 + \bar{b}_1)g_2 \quad (21)$$

$$q \ q : u_1 u_2 + \bar{u}_1 \bar{u}_2 + c_1 c_2 + \bar{c}_1 \bar{c}_2 + d_1 d_2 + \bar{d}_1 \bar{d}_2 + s_1 s_2 + \bar{s}_1 \bar{s}_2 + b_1 b_2 + \bar{b}_1 \bar{b}_2 \quad (22)$$

$$q \ q : u_1 \bar{u}_2 + \bar{u}_1 u_2 + c_1 \bar{c}_2 + \bar{c}_1 c_2 + d_1 \bar{d}_2 + \bar{d}_1 d_2 + s_1 \bar{s}_2 + \bar{s}_1 s_2 + b_1 \bar{b}_2 + \bar{b}_1 b_2 \quad (23)$$

$$\begin{aligned} q \ r : & (u_1 + \bar{u}_1)(c_2 + \bar{c}_2 + d_2 + \bar{d}_2 + s_2 + \bar{s}_2 + b_2 + \bar{b}_2) \\ & + (c_1 + \bar{c}_1)(u_2 + \bar{u}_2 + d_2 + \bar{d}_2 + s_2 + \bar{s}_2 + b_2 + \bar{b}_2) \\ & + (d_1 + \bar{d}_1)(u_2 + \bar{u}_2 + c_2 + \bar{c}_2 + s_2 + \bar{s}_2 + b_2 + \bar{b}_2) \\ & + (s_1 + \bar{s}_1)(u_2 + \bar{u}_2 + c_2 + \bar{c}_2 + d_2 + \bar{d}_2 + b_2 + \bar{b}_2) \\ & + (b_1 + \bar{b}_1)(u_2 + \bar{u}_2 + c_2 + \bar{c}_2 + d_2 + \bar{d}_2 + s_2 + \bar{s}_2) \end{aligned} \quad (24)$$

In case the electro-weak interaction is involved, the foregoing is incorrect and the decomposition is as follows:

$$g \ u : g_1(u_2 + c_2) \quad g \ d : g_1(d_2 + s_2 + b_2) \quad (25)$$

$$g \ u : g_1(\bar{u}_2 + \bar{c}_2) \quad g \ d : g_1(\bar{d}_2 + \bar{s}_2 + \bar{b}_2) \quad (26)$$

$$u \ g : (u_1 + c_1)g_2 \quad d \ g : (d_1 + s_1 + b_1)g_2 \quad (27)$$

$$u : g_1(\bar{u}_1 + \bar{c}_1)g_2 \quad d : g_1(\bar{d}_1 + \bar{s}_1 + \bar{b}_1)g_2 \quad (28)$$

$$u \ u : u_1 u_2 + c_1 c_2 \quad d \ d : d_1 d_2 + s_1 s_2 + b_1 b_2 \quad (29)$$

$$u : u_1 \bar{u}_2 + \bar{c}_1 \bar{c}_2 \quad d : d_1 \bar{d}_2 + \bar{s}_1 \bar{s}_2 + \bar{b}_1 \bar{b}_2 \quad (30)$$

$$u \ u : u_1 \bar{u}_2 + c_1 \bar{c}_2 \quad d \ d : d_1 \bar{d}_2 + s_1 \bar{s}_2 + b_1 \bar{b}_2 \quad (31)$$

$$u : u_1 u_2 + \bar{c}_1 c_2 \quad d : d_1 d_2 + \bar{s}_1 s_2 + \bar{b}_1 b_2 \quad (32)$$

$$u \ d : (u_1 + c_1)(d_2 + s_2 + b_2) \quad u \ d : (u_1 + c_1)(\bar{d}_2 + \bar{s}_2 + \bar{b}_2) \quad (33)$$

$$u : d_1(\bar{u}_1 + \bar{c}_1)(d_2 + s_2 + b_2) \quad u : d_1(\bar{u}_1 + \bar{c}_1)(\bar{d}_2 + \bar{s}_2 + \bar{b}_2) \quad (34)$$

$$d \ u : (d_1 + s_1 + b_1)(u_2 + c_2) \quad d : u_1(\bar{d}_1 + \bar{s}_1 + \bar{b}_1)(u_2 + c_2) \quad (35)$$

$$d : u_1(\bar{d}_1 + \bar{s}_1 + \bar{b}_1)(\bar{u}_2 + \bar{c}_2) \quad d : u_1(\bar{d}_1 + \bar{s}_1 + \bar{b}_1)(\bar{u}_2 + \bar{c}_2) \quad (36)$$

## B The meaning of Esoft

Pre-sampling is performed with KALEU [32], which constructs phase space points from invariants and angles that are generated following pre-defined probability distributions augmented

with adaptive grids, following the method of [31]<sup>1</sup>. The generation of the invariants is such that it mimics the behavior of the squared matrix element as function of the invariants. For example, if the final-state momenta  $p_i, p_j$  belong to on-shell gluons, then the squared matrix element behaves as  $1/s_{ij}$  as function of  $s_{ij} = (p_i + p_j)^2$ . The singularity at  $s_{ij} = 0$  is protected by the phase space cuts, but the squared matrix element will still show a steep behavior as function of  $s_{ij}$ . Therefore, the pre-defined density for the invariant is

$$P_{ij}(s) = \frac{\theta(E_{\text{cm}}^2 - s)}{1 + \log(E_{\text{cm}}^2/E_{\text{soft}}^2)} \left[ \frac{\theta(E_{\text{soft}}^2 - s)}{E_{\text{soft}}^2} + \frac{\theta(s - E_{\text{soft}}^2)}{s} \right]. \quad (37)$$

It increases for decreasing  $s$  until  $s = E_{\text{soft}}^2$ , from where it stays constant, ensuring the coverage of the whole phase space. The exact value of  $E_{\text{soft}}$  does not matter too much, since also a bad choice will be corrected by the adaptive grid. A good choice will, however, help in the efficiency.

---

<sup>1</sup>AVHLIB employs RANLUX [42] for the generation of pseudo random numbers.